Calculator Lab: Improper Integrals

In this lab, we will compare the improper integrals

$$\int_1^{\infty} \frac{1}{x^2} dx \qquad \text{and} \qquad \int_1^{\infty} \frac{1}{\sqrt{x}}.dx$$

(These integrals are called improper because one of the limits of integration is not finite.)

Part I: Numerical Approach

1. (a) Using your calculator, complete the following table:

b	$\int_1^b \frac{1}{x^2} dx$	$\int_1^b \frac{1}{\sqrt{x}}$
10		
100		
1000		
10000		

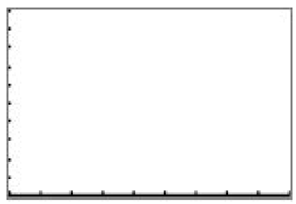
(b) What is happening to $\int_1^b \frac{1}{x^2} dx$ as b gets larger? What should the value of $\int_1^{\infty} \frac{1}{x^2} dx$ be?

(c) What is happening to $\int_1^b \frac{1}{\sqrt{x}}$ as b gets larger? What should the value of $\int_1^{\infty} \frac{1}{\sqrt{x}} dx$ be?

Part II: Graphical Approach

2. Sketch the general shape of $\frac{1}{x^2}$ in the box on the left, and $\frac{1}{\sqrt{x}}$ in the box on the right. (The window dimensions are `xMin` $= 0$, `xMax` $= 10$, `yMin` $= 0$, `yMax` $= 10$.)

3. (a) Graph $\frac{1}{\sqrt{x}}$ with `xMin` $= 100$, `xMax` $= 1000$, `yMin` $= 0$, `yMax` $= 0.1$ in the box below.



i. Approximate the total area of the viewing box.

ii. Approximately what percentage of the viewing box is taken by $\int_{100}^{1000} \frac{1}{\sqrt{x}} dx$?

iii. Using parts (a) and (b) approximate $\int_{100}^{1000} \frac{1}{\sqrt{x}} dx$.

(b) Graph $\dfrac{1}{\sqrt{x}}$ with `xMin` $=$ 10,000, `xMax` $=$ 100,000, `yMin` $=$ 0, `yMax` $=$ 0.01 in the box below.

i. Approximate the total area of the viewing box.

ii. Approximately what percentage of the viewing box is taken by $\displaystyle\int_{10{,}000}^{100{,}000} \frac{1}{\sqrt{x}}dx$?

iii. Using parts (a) and (b) approximate $\displaystyle\int_{10{,}000}^{100{,}000} \frac{1}{\sqrt{x}}dx$.

(c) What would the answers for parts (a) and (b) mean for the value of $\displaystyle\int_{1}^{\infty} \frac{1}{\sqrt{x}}dx$?

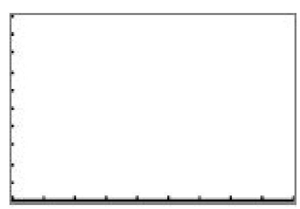
4. (a) Graph $\frac{1}{x^2}$ with $\texttt{xMin} = 10$, $\texttt{xMax} = 100$, $\texttt{yMin} = 0$, $\texttt{yMax} = 0.01$ in the box below.

i. Approximate the total area of the viewing box.

ii. Approximately what percentage of the viewing box is taken by $\int_{10}^{100} \frac{1}{x^2} dx$?

iii. Using parts (a) and (b) approximate $\int_{10}^{100} \frac{1}{x^2} dx$.

(b) Graph $\frac{1}{x^2}$ with $\texttt{xMin} = 100$, $\texttt{xMax} = 1000$, $\texttt{yMin} = 0$, $\texttt{yMax} = 0.0001$ in the box below.



i. Approximate the total area of the viewing box.

ii. Approximately what percentage of the viewing box is taken by $\int_{100}^{1000} \frac{1}{x^2} dx$?

iii. Using parts (a) and (b) approximate $\int_{100}^{1000} \frac{1}{x^2} dx$.

(c) What would the answers for parts (a) and (b) mean for the value of $\int_1^\infty \frac{1}{x^2} dx$?

Part III: Wrap-up

5. Is it true that $\int_0^\infty f(x)dx$ will converge as long as $f(x) \to 0$ when $x \to \infty$?